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Drag force of moving quark at $\mathcal{N} = 2$ supergravity

J. Sadeghi and B. Pourhassan

Sciences Faculty, Department of Physics, Mazandaran University,
P.O. Box 47415-416, Babolsar, Iran
E-mail: pouriya@ipm.ir, b.pourhassan@umz.ac.ir

Abstract: In this paper we consider a moving quark in the thermal plasma at the $\mathcal{N}$=2 Supergravity theory. By using the AdS/CFT correspondence we obtain energy loss of the quark. Then we consider the higher derivative corrections in charged AdS-black hole and calculate the drag force on the moving quark in the thermal plasma. Also we find a limit which $\mathcal{N}$=2 Supergravity solutions are corresponding to the $\mathcal{N}$=4 Super Yang-Mills solutions for the heavy quark.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

Contents

1. Introduction

The relation between gauge theories and string theory has been the subject of many important researches in the last three decades. First, Maldacena in his paper [1] proposed AdS/CFT correspondence, therefore the AdS/CFT correspondence sometimes called Maldacena duality. According to this conjecture there is a relation between a conformal field theory (CFT) in d-dimension and a supergravity theory in $d+1$-dimensional anti de Sitter (AdS) space. Maldacena suggests that a quantum string in $d+1$-dimensional AdS space, mathematically is equivalent to the ordinary quantum field theory with conformal invariance in d-dimensional space-time which lives on the boundary of AdS_{d+1} space. Some details in preliminary formulation of Maldacena are explained by independent works of Witten [2] and Gubser, et al. [3]. An example of AdS/CFT correspondence is the relation between type IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N}=4$ super Yang-Mills gauge theory on the 4-dimensional boundary of AdS_5 space. Today, one of the important issues of researches is using AdS/CFT correspondence in many complicated problem of QCD. For example most of the research is about energy loss of moving charged particles in plasma based on week coupling [4, 3 − 15]. But if one would like to understand the dynamics of such systems in the strong coupling, then there are some complicated calculations in QCD. Therefore energy loss of moving quark through the $\mathcal{N}=4$ super Yang-Mills thermal plasma [16] is studied by using AdS/CFT correspondence [17 − 20]. Furthermore the drag force on a pair of quark-anti quark is considered [17, 21 − 23]. In this way adding the temperature to the system in the gauge theory is corresponding to introduce a black hole (black brane) in the center of AdS_5 space. In this model, at non-zero temperature, one can image open string stretched from D-brane to the horizon and end point of string

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on D-brane represents a quark, so the quark moves and pulls the string. By study the behavior of the string end point, we can obtain energy loss of the quark and drag force in the gauge theory.

In this paper we consider the moving quark in $\mathcal{N}=2$ supergravity thermal medium [24, 25], and calculate drag force in various situations. Indeed, solutions of $\mathcal{N}=2$ supergravity may be solutions of supergravity theory with more supersymmetry ($\mathcal{N}=4$ and $\mathcal{N}=8$). The $\mathcal{N}=2$ supergravity theory in five dimensions can be obtained by compactifying eleven dimensional supergravity in a 3-fold Calabi-Yau [26]. Also anti de Sitter supergravity can obtain by gauging the U(1) subgroup of the SU(2) group in $\mathcal{N}=2$ supersymmetric algebra. Also we would like to add a constant B field to the system and find effect of constant electric and magnetic field on the drag force. Already the drag force in a thermal plasma of $\mathcal{N}=4$ super Yang-Mills theory under the influence of non-zero NSNS B-field background has been studied [27].

Then we consider higher derivative corrections to AdS_5 charged black hole and obtain drag force. Already the higher-derivative curvature corrections to type IIB supergravity was done [28 − 31]. Also effect of curvature-squared corrections on the drag force of moving heavy quark in the $\mathcal{N}=4$ super Yang-Mills plasma is considered by ref. [32]. Furthermore presence of R^2-term in curvature tensor in $\mathcal{N}=2$ supergravity theory has been studied [33]. However, we use the solutions of spherical symmetric AdS_5 charged black hole [24, 25] and find drag force on the moving quark through thermal plasma and then consider the effect of higher derivative terms [34] on the drag force. We note that the stated analysis might be invalid in lower dimension with higher-derivative effective action, without considering proper Kaluza-Klein (K-K) reduction. Therefore universality can be established by carefully analyzing Kaluza-Klein reduction of ten-dimensional action [35].

This paper is organized as follows, in section 2 we review a model for the drag force on quark. Then in section 3, by using solutions of charged AdS-black hole, we find string equation of motion and solve it in three interesting case. In section 4 we discuss the quasi normal modes of string without any external field. In section 5 we add B-field to the system and discuss about the effect of constant electric and magnetic fields, as an external fields, on the drag force. In such cases we follow similar methods used in [17, 20, 27] directly. Finally in section 6 we consider effect of higher derivative terms in our solutions and give some results and summaries in section 7.

2. Drag force

We consider a particle which moves in thermal medium with viscosity, therefore it senses a drag force due to medium. If we consider p as momentum of the particle which moves under external force F and friction coefficient μ, one can write equation of motion as [17],

$$\dot{p} = F - \mu p. \tag{2.1}$$

In order to obtain some information about drag force, it is useful to consider two special cases. First, we assume that momentum of the particle is constant, hence we have $F = \mu p$ and for particle with mass m and non-relativistic momentum $p = mv$ one obtain $\mu m = \frac{F}{v}$.

So, by measurement of velocity of particle for given force we can obtain μm. It shows that we can't find μ independently. The parameter μm is called drag force coefficient.

In the second case we assume that external force does not exist, so from equation of motion (2.1) one can find $p(t) = p(0)e^{-\mu t}$. In other words by measurement of ratio $\frac{\dot{p}}{p}$ or $\frac{\dot{v}}{v}$ we can determine μ without any dependance to m. These lead us to obtain drag force for a moving quark in plasma [17].

The starting point in calculating drag force on quark, is the string action,

$$S = T_0 \int d^2\sigma \mathcal{L}. \tag{2.2}$$

By using the Euler-Lagrange differential equation $\frac{\partial}{\partial \xi} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \xi'} = 0$ with respect to string coordinate ξ and canonical momentum density $\pi_\xi = \frac{\partial \mathcal{L}}{\partial \xi'}$, we have the following equation,

$$\frac{dp}{dt} = \frac{1}{v}\frac{dE}{dt} = \frac{dE}{dx} = -T_0 \pi_\xi, \tag{2.3}$$

this shows that the rate of energy and momentum loss is proportional to the canonical momentum density. The quark with Lagrangian mass m floats in the thermal plasma with temperature T, and it has a rest mass $M_{\text{rest}}(T)$ which is different from physical (Lagrangian) mass of the quark, because the temperature effect on the mass. On the other hand if the quark moves in the thermal plasma then it will have a kinetic mass $M_{\text{kin}}(T)$, so we have [17],

$$E = M_{\text{rest}}(T) + \frac{p^2}{2M_{\text{kin}}(T)}. \tag{2.4}$$

Difference of $M_{\text{kin}}(T)$ and $M_{\text{rest}}(T)$ for heavy quark ($m \gg \Delta m(T)$) is negligible, so we have,

$$M_{\text{kin}}(T) = M_{\text{rest}}(T) + \mathcal{O}\left[m\left(\frac{\triangle m(T)}{m}\right)^2\right], \tag{2.5}$$

where $\Delta m(T)$ is thermal mass shift. By knowledge about the drag force and mass we can obtain diffusion coefficient for non-relativistic quark, which depends to temperature and drag force [17],

$$D = \frac{T}{m\mu}, \tag{2.6}$$

where parameter D is a phenomenological property.

3. String equation of motion

As we know the temperature in the supersymmetric gauge theory is equal to existence a black hole with a flat horizon in the anti de Sitter space. As mentioned [33] Lagrangian for the bosonic sector of pure $\mathcal{N}$=2 gauged supergravity in five dimensions is,

$$e^{-1}\mathcal{L}_0 = R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 12\Lambda^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda. \tag{3.1}$$

Then the AdS_5 black hole solutions, given by [24, 25],

$$\begin{aligned} ds^2 &= -\frac{f}{H^2}dt^2 + H\left(r^2dx^2 + \frac{dr^2}{f}\right), \\ A &= \sqrt{3}\coth\beta\left(\frac{1}{H} - 1\right)dt, \\ f &= 1 - \frac{\eta}{r^2} + \Lambda^2 r^2 H^3, \\ H &= 1 + \frac{\eta\sinh^2\beta}{r^2}, \end{aligned} \tag{3.2}$$

where r is axis along the black hole, so the horizon of black hole is in $r = r_h$. Also we assume that the motion in sphere S^5 is only in transverse axis x. The β parameter is related to the electric charge of black hole and Λ is cosmological constant, also expansion parameter η is called non-extremality parameter.

The dynamics of a open string is described by the following Nambo-Goto action,

$$S = -T_0\int d\tau d\sigma\sqrt{-g}, \tag{3.3}$$

where $-g = -det g_{ab}$, g_{ab} is the metric of string worldsheet, and space-time metric $G_{\mu\nu}$ is given by line element (3.2). As we told the string moves only in x direction and we use static gauge so that we take $\tau = t$ and $\sigma = r$. Therefore string world-sheet described by $x(t, r)$, so we can write,

$$-g = \frac{1}{H} - \frac{H^2r^2}{f}\dot{x}^2 + \frac{fr^2}{H}x'^2, \tag{3.4}$$

and the equation of motion is,

$$\frac{\partial}{\partial r}\left(\frac{fr^2}{H\sqrt{-g}}x'\right) - \frac{H^2r^2}{f}\frac{\partial}{\partial t}\left(\frac{\dot{x}}{\sqrt{-g}}\right) = 0. \tag{3.5}$$

Here, we can obtain component of canonical momentum density for $\mu = x, r, t$ as,

$$\begin{aligned} \pi_t^0 &= -\frac{T_0}{\sqrt{-g}}\frac{(1 + fr^2x'^2)}{H}, \\ \pi_x^0 &= \frac{T_0}{\sqrt{-g}}\frac{H^2r^2}{f}\dot{x}, \\ \pi_r^0 &= -\frac{T_0}{\sqrt{-g}}\frac{H^2r^2}{f}\dot{x}x', \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} \pi_t^1 &= \frac{T_0}{\sqrt{-g}}\frac{fr^2}{H}\dot{x}x', \\ \pi_x^1 &= -\frac{T_0}{\sqrt{-g}}\frac{fr^2}{H}x', \\ \pi_r^1 &= \frac{T_0}{\sqrt{-g}}\left(-\frac{1}{H} + \frac{H^2r^2}{f}\dot{x}^2\right). \end{aligned} \tag{3.7}$$

Then the total energy and momentum of string can be obtained by following relations,

$$E = -\int dr \pi_t^0,$$
$$p = \int dr \pi_x^0. \tag{3.8}$$

We note that there is interesting limit where above relations (10-13) are corresponding to the case of a heavy quark moving through $\mathcal{N}$=4 supersymmetric Yang-Mills plasma. That is $\eta \to 0$ limit where we have $H = 1$, then by rescaling $r = L^2 u$ (L is AdS radius) and setting $f(r) \equiv h(u)$ (in 4-dimension we know $h = u^2[1 - (\frac{u_h}{u})^4]$, where u_h is position of black hole horizon) our results reduce to the ref. [17].

The Hawking temperature of the solutions (3.2) is given by [36],

$$T = \frac{2 + 3k - k^3}{2(1+k)^{\frac{3}{2}}} \frac{r_h}{\pi L^2}, \tag{3.9}$$

where $k \equiv \frac{\eta \sinh^2\beta}{r_h^2}$. Clearly at the $\eta \to 0$ limit we have $T = \frac{r_h}{\pi L^2}$, which is agree with Hawking temperature of the black hole for $d = 4$ [17]. As we know the consideration of a quark in the gauge theory is corresponding to string in AdS space, also spiking about temperature of plasma is corresponding to existence of a black hole in AdS space. Similarly to this, the consideration quark flavor in the field theory is corresponding to adding a D-brane. D-brane covers an sphere S^3 in S^5 which has minimum radius r_m.

Now we come back to string equation of motion. In order to solve the equation of motion (3.5) and obtain drag force, we consider three cases. First of all, we take an static string which is corresponding to the rest quark. In the second case we assume that the straight string moves with the constant velocity. Finally in the third case, for physical motion, we consider curved moving string with constant speed v.

A quark in the $\mathcal{N}$=2 supergravity thermal plasma is equal to the string which stretched from D-brane to the black hole horizon. End point of string on D-brane represents the quark.

In this case the simplest solution which satisfy equation of motion (3.5) is $x(r,t) = x_0$. It describes an static string stretched straightforwardly from $r = r_m$ on D-brane to $r = r_h$ in the black hole horizon, and represents the rest quark in thermal plasma clearly. Therefore by using equations (3.6) and (3.8), we can calculate total energy and momentum. Momentum of the rest quark vanishes,$\pi_x^0 = \pi_r^0 = \pi_t^1 = \pi_x^1 = 0$, as we expected and the total energy will be as,

$$E = T_0 \left(r_m \sqrt{1 + \frac{\eta \sinh^2\beta}{r_m^2}} - r_h \sqrt{1 + \frac{\eta \sinh^2\beta}{r_h^2}} \right). \tag{3.10}$$

As we see in equation (3.9) the temperature of black hole is zero at $r_h^2 = \frac{\eta \sinh^2\beta}{2}$. There are also another solutions such as $r_h^2 = 0$ and $r_h^2 = -\eta \sinh^2\beta$ which have singularity, therefore they are not allowed. Again we note that at the limit of $\eta \to 0$ the temperature is proportional to r_h and the zero temperature limit is equal to $r_h \to 0$ [17]. However we know that the extremal limit of background (3.2) is obtained in $\eta \to \infty$ and $\beta \to 0$ limit [34]. On the other hand in zero temperature we can interpret E as physical (Lagrangian) mass of quark. Hence, in limit of zero temperature we can write,

$$E = m = T_0 \left(r_m \sqrt{1 + \frac{\eta \sinh^2 \beta}{r_m^2}} - \sqrt{\frac{3\eta \sinh^2 \beta}{2}} \right). \tag{3.11}$$

By increasing the radius, the mass of quark increases. So that if D-brane moves to boundary of AdS space ($r \to \infty$) the mass of quark will be infinite. But the increasing of the temperature effect on the relation between Lagrangian mass and r_m. The energy E in (3.10) at non-zero temperature interpreted as free energy of the rest quark in $\mathcal{N}$=2 supergravity thermal plasma, then one can obtain the relation between free energy and the thermal rest mass of quark. So we follow from used techniques in [17] and summarize some results in table 1.

Now we assume that the straight string moves with the speed of v, so $x(r,t) = x_0 + vt$ may be solution of equation (3.5). In this case one can find non-zero components of the canonical momentum density as,

$$\begin{aligned} \pi_t^0 &= -\frac{T_0}{H\sqrt{\frac{1}{H} - \frac{H^2 r^2}{f} v^2}}, \\ \pi_x^0 &= \frac{T_0 H^2 r^2 v}{f\sqrt{\frac{1}{H} - \frac{H^2 r^2}{f} v^2}}, \\ \pi_r^1 &= -T_0 \sqrt{\frac{1}{H} - \frac{H^2 r^2}{f} v^2}. \end{aligned} \tag{3.12}$$

We see that the energy and x-component of momenum densities on the string worldsheet are non-zero, so one can obtain total energy and momentum of the string, but we don't like to perform this step for following reason. As we saw, $\eta \to 0$ limit of the $\mathcal{N}$=2 supergravity theory is corresponding to the $\mathcal{N}$=4 super yang-mills theory in the problem of drag force, so the moving straight string is not a physical motion, because in that case the square root quantity in the action is negative and we have imaginary action, energy and momentum. Therefore, we leave this state and take curved moving string with speed of v as physical motion, for such system, one can choose,

$$x(r,t) = x(r) + vt, \tag{3.13}$$

as solution of string equation (3.5). First we try to determine $x(r)$. It is clear that $\dot{x}$, x' and $\sqrt{-g}$ are independent of time, therefore from equation of motion (3.5) we have,

$$x' = \frac{Hv\sqrt{-g}C}{fr^2}, \tag{3.14}$$

where C is a constant and,

$$-g = \frac{1}{H} - \frac{H^2 r^2}{f} v^2 + \frac{fr^2}{H} x'^2. \tag{3.15}$$

Here, we use the equations (3.7) and (3.14), and obtain momentum density as,

$$\begin{aligned} \pi_t^1 &= T_0 C v^2, \\ \pi_x^1 &= -T_0 C v. \end{aligned} \tag{3.16}$$

As we see this two currents are constant along the string. Now by using the equation (3.14) and (3.15) we have,

$$-g = \frac{r^2}{H} \frac{f - H^3 r^2 v^2}{f r^2 - C^2 v^2 H}. \tag{3.17}$$

The important problem here is that $-g$ must be positive. By appropriate choice of constant C, we have $-g > 0$ everywhere. So one can obtain,

$$C = \pm\left(1 + \frac{\eta \sinh^2\beta}{r_c^2}\right) r_c^2, \tag{3.18}$$

where critical radius r_c is root of following equation,

$$r^2 - \eta + (\Lambda^2 - v^2) r^4 \left(1 + \frac{\eta \sinh^2\beta}{r^2}\right)^3 = 0. \tag{3.19}$$

Furthermore, by using C from equation (3.18) in equation (3.16) the energy current into the horizon is,

$$\pi_t^1 = T_0 v^2 \left(1 + \frac{\eta \sinh^2\beta}{r_c^2}\right) r_c^2, \tag{3.20}$$

and the momentum current into the horizon is,

$$\pi_x^1 = -T_0 v \left(1 + \frac{\eta \sinh^2\beta}{r_c^2}\right) r_c^2. \tag{3.21}$$

Then from equation (3.6) we obtain the π_t^0 and π_x^0 as follow,

$$\begin{aligned} \pi_t^0 &= -\frac{T_0}{\left(1 + \frac{\eta \sinh^2\beta}{r^2}\right)} \sqrt{\frac{\left(1 + \frac{\eta \sinh^2\beta}{r_c^2}\right)^2 r_c^4}{\left(1 + \frac{\eta \sinh^2\beta}{r_h^2}\right) r_h^4}} \left[1 + \frac{\left(\left(1 + \frac{\eta \sinh^2\beta}{r^2}\right) v \sqrt{(1 + \frac{\eta \sinh^2\beta}{r_h^2}) r_h^4}\right)^2}{\left(1 - \frac{\eta}{r^2} + \Lambda^2 r^2 \left(1 + \frac{\eta \sinh^2\beta}{r^2}\right)^3\right) r}\right] \\ \pi_x^0 &= T_0 \sqrt{\frac{\left(1 + \frac{\eta \sinh^2\beta}{r_c^2}\right)^2 r_c^4}{\left(1 + \frac{\eta \sinh^2\beta}{r_h^2}\right) r_h^4}} \frac{\left(1 + \frac{\eta \sinh^2\beta}{r^2}\right)^2 r^2 v}{1 - \frac{\eta}{r^2} + \Lambda^2 r^2 \left(1 + \frac{\eta \sinh^2\beta}{r^2}\right)^3}, \end{aligned} \tag{3.22}$$

in order to obtain the total energy and momentum of string one can integrate them over the worldsheet from r_h to r_m. We note that for $\eta \to 0$ limit, the critical radius is $r_c^2 = \frac{1}{v^2 - \Lambda^2}$, therefore equation (3.18) reduces to,

$$C = \pm \frac{1}{v^2 - \Lambda^2}, \tag{3.23}$$

and we have,

$$-g = r_h^4 (v^2 - \Lambda^2)^2. \tag{3.24}$$

We saw that loss of the energy current of quark by the string, π_t^1, is proportional to C (see equation (3.16)). So the positive sign in C shows the energy current from quark to the horizon along string. But the negative sign of C shows energy current from the horizon to quark, in this case the string moves and pull the quark, so it is non-physical situation, therefore we give positive sign in equations (3.18) and (3.23) for constant C. Hence by using equations (3.14), (3.23) and (3.24) we obtain x' as,

$$x' = \frac{v r_h^2}{(1+\Lambda^2 r^2) r^2}. \tag{3.25}$$

Here we should strongly recommend that at $\eta \to 0$ limit one can compare presence problem to the quark moving through $\mathcal{N} = 4$ SYM plasma. As mentioned already we have correspondence between $\mathcal{N} = 2$ supergravity and $\mathcal{N} = 4$ super Yang-Mills Theories. Hence by integrating from x' with respect to r, we find string coordinate (3.13) as,

$$x(r,t) = x_0 + vt + v r_h^2 \left[\frac{\pi}{2} - \Lambda \tan^{-1}(\Lambda r) - \frac{1}{r}\right], \tag{3.26}$$

which, other than a constant coefficient, is different from [17] only in the last term. It is due to difference between $f(r)$ in our paper and $h(u)$ in $\mathcal{N} = 4$ SYM. The energy and momentum current to the black hole horizon from equation (3.16) give us the following equations,

$$\begin{aligned}\pi_t^1 &= \frac{T_0 v^2}{v^2 - \Lambda^2},\\ \pi_x^1 &= -\frac{T_0 v}{v^2 - \Lambda^2}.\end{aligned} \tag{3.27}$$

Now we would like to interpret solutions of stationary string as describing the steady-state behavior of a moving quark through the $\mathcal{N} = 2$ supergravity medium under effect of a constant electric field ε. The velocity of quark is going to an equilibrium value v where the drag force on quark is equal to the external force due to the constant electric field. The constant electric field ε is a real parameter. That is a U(1) gauge which coupled to the brane. The work of electric field on the quark is proportional to its velocity i.e. $\varepsilon \cdot v$. This work is equal to the energy loss of quark in the plasma. According to the relation (3.27) the energy current of the string through the horizon is,

$$\pi_t^1 = -\pi_x^1 v = T_0 v^2 \left(1 + \frac{\eta \sinh^2 \beta}{r_c^2}\right) r_c^2, \tag{3.28}$$

where in $\eta \to 0$ limit reduces to the $\pi_t^1 = \frac{T_0 v^2}{v^2 - \Lambda^2}$. Therefore, one can write equation $\pi_x^1 = -\varepsilon$ and find,

$$\mu m = T_0 \left(1 + \frac{\eta \sinh^2 \beta}{r_c^2}\right) r_c^2, \tag{3.29}$$

where we assume that the quark is an excitation mode of string with mass m and non-relativistic momentum $p = mv$, so we can gave the rate of transferred momentum π_x^1 equal to momentum loss of quark $\dot{p} = -\mu p$. Therefore, at the $\eta \to 0$ limit for heavy quark (low velocity) one can obtain, $\dot{p} = \frac{T_0}{\Lambda^2}$. Indeed, $\frac{dp}{dt} = -\pi_x^1$ and $\frac{dE}{dt} = \pi_t^1$ are the rate of momentum and energy, respectively, which transfer to the quark by the electric field. By using non-relativistic momentum $p = mv$, one can write the diffusion coefficient for the quark as,

$$D = \frac{T}{T_0}\left[\left(1+\frac{\eta\sinh^2\beta}{r_c^2}\right)r_c^2\right]^{-1}. \tag{3.30}$$

As we see, information about the drag force is equivalent to information about the diffusion coefficient of quark. One can take $\eta \to 0$ limit in the above expression and neglect the squared-velocity terms for heavy quark and obtain $D = -\frac{T}{T_0}\Lambda^2$.

4. Small fluctuations

In this section we again consider a quark moving in the $\mathcal{N}$=2 supergravity thermal plasma without any external field. The aim of this section is studying behavior of string after long time and with slow velocity. We consider the dynamics of such a system at a long time as an small perturbation in the static string which describes the rest of quark. Therefore, we have quasinormal modes of string worldsheet which are small fluctuations in the string. This small fluctuations around the string means that $\dot{x}$ and x' in string equation of motion are small. Hence, we neglect $\dot{x}^2$ and x'^2 in $-g$, this leads us to $-g = \frac{1}{H}$, then equation of motion (3.5) reduces to,

$$\frac{\partial}{\partial r}\left(\frac{fr^2}{\sqrt{H}}x'\right) - \frac{H^{\frac{5}{2}}r^2}{f}\ddot{x} = 0. \tag{4.1}$$

Now we try to solve this problem for special case with the time dependance $e^{-\mu t}$. In that case we put,

$$x(r,t) = x(r)e^{-\mu t}, \tag{4.2}$$

to equation (4.1) which yield to eigenvalue equation,

$$Ox = \mu^2 x, \tag{4.3}$$

where

$$O = \frac{f}{H^{\frac{5}{2}}r^2}\frac{d}{dr}\frac{fr^2}{\sqrt{H}}\frac{d}{dr}. \tag{4.4}$$

Eigenvalue equation (4.3) at $\eta \to 0$ limit with $H = 1$ and by $\Lambda^2 = -1$ reduces to associated Legendre equation which have solutions in terms of hypergeometric functions. In other words at such limit, our solutions are similar to solutions of the following model, heavy quark moving through $\mathcal{N}$=4 super Yang-Mills thermal plasma in 2 dimension [17]. As we expected before, the calculation of drag force in the $\mathcal{N}$=2 supergravity theory at $\eta \to 0$ limit is corresponding to the heavy quark in $\mathcal{N}$=4 super Yang-Mills theory.

In order to obtain $x(r)$ we expand it in terms of powers μ and follow directly from [17], also take $\eta \to 0$ to obtain,

$$\begin{aligned} x_1'(r) &= \frac{-A}{(1+\Lambda^2r^2)r^2}, \\ x_2'(r) &= \frac{A}{\Lambda^2}\left[\frac{1}{(1+\Lambda^2r^2)r} - \frac{\pi}{2\Lambda(1+\Lambda^2r^2)r^2}\right], \end{aligned} \tag{4.5}$$

where A ia an arbitrary constant. Then by using Neumann boundary condition $x'(r_m) = 0$ one can obtain friction coefficient μ as a following,

$$\mu = \frac{\Lambda^2}{r_m - \frac{\pi}{2\Lambda}}. \tag{4.6}$$

It is the lowest eigenvalue of operator O or equivalently lowest quasinormal modes of the string. The exponential form of x at the end point of string leads us to have $\dot{v} = -\mu v$, so, by using the relation $m = T_0 r_m$ at the $\eta \to 0$ limit one can obtain drag force as,

$$\frac{dp}{dt} = -\frac{T_0 \Lambda^2 r_m v}{r_m - \frac{\pi}{2\Lambda}}. \tag{4.7}$$

Finally we obtain diffusion coefficient of the quark as,

$$D = \frac{T}{m\Lambda^2}\left(\frac{m}{T_0} - \frac{\pi}{2\Lambda}\right). \tag{4.8}$$

Now by using equation (4.1) we will find total energy and momentum of string. First, by using relation (3.6) and $-g \simeq \frac{1}{H}$ at slow velocity we find,

$$\pi_x^0 = -\frac{T_0}{\mu}\left(\frac{fr^2}{\sqrt{H}}x'\right)', \tag{4.9}$$

in that case we have used from $x(r,t) = x(r)e^{-\mu t}$. Thus by integrating from equation (4.9) and by Neumann boundary condition we easily obtain the total momentum as follows,

$$p = \frac{T_0}{\mu}\lambda(\Lambda, \beta, \eta)x'(r_{\min}), \tag{4.10}$$

where,

$$\lambda(\Lambda, \beta, \eta) = \left[\frac{r_{\min}^2 + \Lambda^2 r_{\min}^4\left(1 + \frac{\eta \sinh^2\beta}{r_{\min}^2}\right)^3 - \eta}{\sqrt{1 + \frac{\eta \sinh^2\beta}{r_{\min}^2}}}\right]. \tag{4.11}$$

and at $\eta \to 0$ limit, $-g \to 1$ we have,

$$p = \frac{T_0}{\mu}(1 + \Lambda^2 r_{\min}^2) r_{\min}^2 x'(r_{\min}), \tag{4.12}$$

We see that at $\eta \to 0$ limit the parameters β and η play no role and we have $\lambda(\Lambda) = 1 + \Lambda^2 r_{\min}^2$ in equation (4.12).

In order to obtain the total energy we expand $\frac{1}{\sqrt{-g}}$ to second order of speeds, then by using equation of motion we have,

$$\pi_t^0 = -T_0\left[\frac{1}{\sqrt{H}} + \frac{1}{2}\left(\frac{fr^2}{\sqrt{H}}xx'\right)'\right]. \tag{4.13}$$

Here, by using Neumann boundary condition and integration on the equation (4.13) we have,

$$E = T_0\left[\sqrt{r_m^2 + \eta \sinh^2\beta} - \sqrt{r_{\min}^2 + \eta \sinh^2\beta} - \frac{1}{2}\lambda(\Lambda, \beta, \eta)x(r_{\min})x'(r_{\min})\right], \tag{4.14}$$

which reduces to the following equation at the $\eta \to 0$ limit,

$$E = T_0 \left[r_m - r_{\min} - \frac{1}{2}(1 + \Lambda^2 r_{\min}^2) r_{\min}^2 x(r_{\min}) x'(r_{\min}) \right]. \tag{4.15}$$

Let's take $p = M\dot{x}$ as momentum of particle with effective mass M and $x(r,t) = x(r)e^{-\mu t}$, so the equations (4.12) and (4.15) give us the following equation,

$$E = T_0(r_m - r_{\min}) + \frac{p^2}{2M}. \tag{4.16}$$

This is a well known relation between energy and momentum. By comparing Equation (4.16) and (2.4) one can interpret $T_0(r_m - r_{\min})$ as $M_{\rm rest}$ and M as $M_{\rm kin}$ of the quark.

5. Adding B-field

In this section, we consider a moving quark with constant velocity v through $\mathcal{N}$=2 supergravity thermal plasma, and introduce a constant B-field including electric field E and magnetic field $\mathcal{H}$ along the x^1 and x^2 direction. Therefore one can couple B-field to line elements (3.2) as following,

$$\begin{aligned} ds^2 &= -\frac{f}{H^2}dt^2 + H\left(r^2 d\vec{x}^2 + \frac{dr^2}{f}\right) \\ B &= E dt \wedge dx_1 + \mathcal{H} dx_1 \wedge dx_2, \end{aligned} \tag{5.1}$$

where we have introduced the NS-NS constant antisymmetric fields $B_{01} = E$ and $B_{12} = \mathcal{H}$. All of other components of B-field are zero. Now we may describe such string attached to the quark by the following solutions,

$$\begin{aligned} x_1(t,r) &= x_1(r) + v_1 t, \\ x_2(t,r) &= x_2(r) + v_2 t, \\ x_3(t,r) &= 0, \end{aligned} \tag{5.2}$$

it means that string motion is in plan (x_1, x_2), so we have velocity $\vec{\dot{x}} = (v_1, v_2)$, and projected direction of string tail $\vec{x}' = (x_1', x_2')$ which have different directions, these may be in opposite directions or perpendicular to each other. As before, we choose static gauge $\tau = t$ and $\sigma = r$, thus, the Lagrangian density is given by,

$$\mathcal{L} = -\sqrt{\frac{1}{H} - \frac{H^2 r^2}{f}\vec{v}^2 + \frac{f r^2}{H}\vec{x}'^2 - (E x_1' + \mathcal{H}(\vec{v} \times \vec{x}'))^2}. \tag{5.3}$$

We continue our study in two cases of electric field and magnetic field separately.

First, we consider a moving quark with constant speed of v and constant electric field $E = B_{01}$. To study the effects of E, we choose the moving direction of the quark in the x^1 direction. Therefore we have a more restricted solution than the solutions (5.2) as following,

$$\begin{aligned} x_1(t,r) &= x_1(r) + vt, \\ x_2(t,r) &= x_3(t,r) = 0. \end{aligned} \tag{5.4}$$

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Now, by using Lagrangian density, we can calculate equation of motion and $x_1(r)$ easily. Then x_1-component of momentum density obtained by relation $\pi_{x_1} = \frac{\partial\mathcal{L}}{\partial x_1'}$. Finally by using the reality and physical motion condition for $x_1(r)$ one can obtain,

$$\pi_{x_1} = \left(\frac{f(r_c)r_c^2}{H(r_c)} - E^2\right)^{\frac{1}{2}}, \tag{5.5}$$

where critical radius r_c is the root of equation (3.19). So at the $\eta \to 0$ limit ($H = 1$) where the critical radius becomes $r_c^2 = \frac{1}{v^2-\Lambda^2}$, one can obtain the drag force as,

$$\frac{dp}{dt} = -T_0\left(\frac{v^2}{(v^2-\Lambda^2)^2} + E^2\right)^{\frac{1}{2}}, \tag{5.6}$$

which is coincide with [27] if we set $\Lambda^2 = 1$ and $1 - v^2 \equiv \frac{L^4}{r_h^4}$. In equation (5.6) we have non-perturbation nature for the large E, however it may be interesting to consider small E. In this case we expand square root in equation (5.6) and find,

$$\frac{dp}{dt} = -T_0\frac{v}{(v^2-\Lambda^2)}\left(1 + \frac{(v^2-\Lambda^2)^2}{2v^2}E^2 + \mathcal{O}(E^4)\right). \tag{5.7}$$

Here, we note that the corrections are in terms of even power of E. The effect of electric field on drag force is in the form of $T_0\frac{\Lambda^2-v^2}{2v}E^2$ for small E. If $v > 0$ and $v^2 > \Lambda^2$ or $v < 0$ and $v^2 < \Lambda^2$ then the additional term due to electric field is opposite to drag force, instead if $v < 0$ and $v^2 > \Lambda^2$ or $v > 0$ and $v^2 < \Lambda^2$ then electric field increases the drag force. Also if E^2 terms vanished, we reproduce same results given in refs. [17, 20].

Second, we consider the following solutions,

$$\begin{aligned} x_1(t,r) &= x_1(r) + vt, \\ x_2(t,r) &= x_2(r), \\ x_3(t,r) &= 0. \end{aligned} \tag{5.8}$$

Indeed we have $\mathcal{H} = B_{12}$ and $E = 0$. Therefore, we have two momentum density π_{x_1} and π_{x_2} conjugate with coordinates x_1 and x_2 respectively. In this case one can find,

$$\begin{aligned} x_1' &= \pi_{x_1}\sqrt{\frac{\beta\left(\frac{1}{H} - \frac{H^2r^2}{f}v^2\right)}{\frac{fr^2}{H}\left[\left(\pi_{x_1}^2 - \frac{fr^2}{H}\right)\left(\pi_{x_2}^2 - \beta\right) - \pi_{x_1}^2\pi_{x_2}^2\right]}}, \\ x_2' &= \pi_{x_2}\sqrt{\frac{\frac{fr^2}{H}\left(\frac{1}{H} - \frac{H^2r^2}{f}v^2\right)}{\beta\left[\left(\pi_{x_1}^2 - \frac{fr^2}{H}\right)\left(\pi_{x_2}^2 - \beta\right) - \pi_{x_1}^2\pi_{x_2}^2\right]}}, \end{aligned} \tag{5.9}$$

where $\beta = \frac{fr^2}{H} - v^2\mathcal{H}^2$. The square root quantity is positive for critical radius r_c obtained by equation (3.19), then, with respect to the solutions (5.8) we can find,

$$\begin{aligned} \pi_{x_1}^2 &= \left(1 + \frac{\eta\sinh^2\beta}{r_c^2}\right)^2 r_c^4 v^2, \\ \pi_{x_2}^2 &= 0, \end{aligned} \tag{5.10}$$

which agrees with equation (3.28) where we obtained momentum density without external field. Therefore one can obtain the drag forces as,

$$\frac{dp_1}{dt} = -T_0\left(1+\frac{\eta\sinh^2\beta}{r_c^2}\right)r_c^2 v,$$
$$\frac{dp_2}{dt} = 0, \tag{5.11}$$

this means that there is no drag force along x_2-direction and the B_{12} has no effect on the motion along v_1. Indeed $\frac{dp_2}{dt}=0$ agrees with a vanishing v_2 in solutions (5.8).

At the $\eta\to 0$ limit, where $H=1$, we have $r_c^2=\frac{1}{v^2-\Lambda^2}$ and $\beta(r_c)=v^2(\frac{1}{v^2-\Lambda^2}-\mathcal{H})$. In this case one can find the drag forces as,

$$\frac{dp_1}{dt} = -T_0\frac{v}{v^2-\Lambda^2},$$
$$\frac{dp_2}{dt} = 0. \tag{5.12}$$

Now we would like to consider the case of $\vec{v}\perp E$, where we have the following solutions,

$$x_1(t,r) = x_1(r),$$
$$x_2(t,r) = x_2(r)+vt,$$
$$x_3(t,r) = 0, \tag{5.13}$$

instead solutions (5.8). We discuss at the $\eta\to 0$ limit, so, in this case we have the same analysis as above, but here there are two solutions. The first solution gives $\pi_{x_1}=0$ and $\pi_{x_2}=\sqrt{\beta(r_c)}$. So we have,

$$\frac{dp_1}{dt} = 0,$$
$$\frac{dp_2}{dt} = -T_0 v\sqrt{\frac{1}{v^2-\Lambda^2}-\mathcal{H}}. \tag{5.14}$$

Therefore we have no drag force along the moving direction. Then, the second solution gives $\pi_{x_1}=\frac{v}{v^2-\Lambda^2}$ and $\pi_{x_2}=0$. Hence we obtain the drag force same as in equation (5.12). In this case the electric field E has no effect on the drag force.

6. Higher derivative corrections

The lowest order in the string length leads us to having an expansion of the effective action in power of α'. Higher powers of α' correspond to higher derivative terms provide information about naturally stringy effects, and knowing them leads to some applications. The higher order corrections are depend to the physics of black hole. For instance, they are relevant to the resolution of singularities of the black hole. They are also significant for stretching the horizon if solution of the classical black hole does not have one. They also yield to a modification of the Beckenstein-Hawking area law for the entropy [37]. Therefore

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higher derivative corrections allows us to the better understand AdS/CFT correspondence. As we know in the ref. [34] the higher derivative Lagrangian is,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{R^2} + \mathcal{L}_{F^4} + \mathcal{L}_{RF^2}, \tag{6.1}$$

where $\mathcal{L}_0$ is given in equation (3.1) and the additional terms are,

$$\begin{aligned} e^{-1}\mathcal{L}_{R^2} &= \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}, \\ e^{-1}\mathcal{L}_{F^4} &= \beta_1 (F_{\mu\nu}F^{\mu\nu})^2 + \beta_2 F^{\mu}_{\nu}F^{\nu}_{\rho}F^{\rho}_{\sigma}F^{\sigma}_{\mu}, \\ e^{-1}\mathcal{L}_{RF^2} &= \gamma_1 R F_{\mu\nu}F^{\mu\nu} + \gamma_2 R_{\mu\nu}F^{\mu\rho}F^{\nu}_{\rho} + \gamma_3 R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}. \end{aligned} \tag{6.2}$$

Here we consider the first order corrections to the R-charged black hole solution of the equation (3.2) in ref. [34]. They have considered the coefficients $(\alpha_1, \alpha_2, \cdots, \gamma_3)$ of the four-derivative terms in equation (6.2) as small parameters. So, the first order corrections of charged black hole solution (3.2) will be the following,

$$\begin{aligned} A &= \sqrt{3}\coth\beta\left(\frac{1+a_1}{H} - 1\right)dt, \\ f &= 1 - \frac{\eta}{r^2} + \Lambda^2 r^2 H^3 + f_1 = f_0 + f_1, \\ H &= 1 + \frac{\eta \sinh^2\beta}{r^2} + h_1 = h_0 + h_1, \end{aligned} \tag{6.3}$$

where a_1, f_1 and h_1 are small corrections obtained in ref. [34]. We are going to apply the first order corrections of charged black hole solution to drag force of quark, hence for the moving quark through the thermal plasma with constant speed v one can write,

$$\mathcal{L} = -\sqrt{-g} = -\sqrt{\frac{1}{h_0+h_1} - \frac{(h_0+h_1)^2 r^2}{(f_0+f_1)}v^2 + \frac{(f_0+f_1)r^2}{(h_0+h_1)}x'^2}. \tag{6.4}$$

Similar to previous sections one can calculate drag force as,

$$\frac{dp}{dt} = -T_0 r_c^2 v (h_0 + h_1). \tag{6.5}$$

where r_c is root of equation,

$$\frac{1}{h_0+h_1} - \frac{(h_0+h_1)^2 r^2}{(f_0+f_1)}v^2 = 0. \tag{6.6}$$

To obtain expression (6.5) we assume that $r \gg \eta$ but non-zero, so one can neglect second powers of η in correction terms (a_1, h_1 and f_1). In that case we have straightforward generalization of relation (3.29) by replacing $H = h_0$ by $H = h_0 + h_1$. Here, the critical radius at the $\eta \to 0$ limit reduces to the following equation,

$$r_c^2 = \frac{1}{v^2 - \Lambda_{\text{eff}}^2}, \tag{6.7}$$

where

$$\Lambda_{\text{eff}}^2 = \Lambda^2\left[1 + \frac{2}{3}(10\alpha_1 + 2\alpha_2 + \alpha_3)\Lambda^2\right], \tag{6.8}$$

which α_1, α_2 and α_3 are coefficients related to the higher derivative corrections [34]. These are specified by the underlying theory. So that the the α_1 and α_2 coefficients are not fixed if we haven't an off-mass-shell formulation such as string field theory, and they may be vanished by an on-mass-shell field redefinition [34]. In this way, we obtain physical information from the underlying string theory just by the α_3 coefficient. Thus, the drag force on moving quark in the $\mathcal{N}$=2 supergravity thermal plasma at the $\eta \to 0$ limit and under higher derivative corrections is,

$$\frac{dp}{dt} = -T_0 \frac{v}{v^2 - \Lambda_{\rm eff}^2}. \tag{6.9}$$

So the drag force coefficient (μm) is given by $\frac{T_0}{v^2-\Lambda_{\rm eff}^2}$. Finally we can obtain diffusion coefficient for the heavy quark as same as section 3, but the cosmological constant Λ must replaced by the effective cosmological constant $\Lambda_{\rm eff}$. Similar to this, we can generalize drag force in presence of electric and magnetic fields, so direct extension of equations (5.6), (5.12) and (5.14) under higher derivative corrections is given by replacing Λ whit $\Lambda_{\rm eff}$.

7. Conclusion

By using AdS/CFT correspondence we studied the drag force on moving quark through $\mathcal{N}$=2 supergravity thermal plasma. We used the solutions of AdS_5 charged black hole and obtained components of energy and momentum densities of string in three cases; static string, moving straight string and moving curved string. We have shown that the only physical state with constant velocity is curved string. Also we discussed about quasinormal modes of string and found drag force by lowest quasinormal modes of static string which was accelerated. Then we considered B-field in the thermal plasma and obtained the effect of constant electric field and magnetic field on the drag force.

The interesting point and important result we obtained in this article is that the limit of $\eta \to 0$ is corresponding to the results of ref. [17], where authors calculated energy loss of heavy quark through $\mathcal{N}$=4 super Yang-Mills thermal plasma by using AdS/CFT correspondence. Therefore, we summarized corresponding parameter in table 1. So that at $\eta \to 0$ limit i.e. $H = 1$ and with rescaling $r = L^2 u$ and setting $f \equiv h$ for heavy quark both results of drag force in $\mathcal{N}$=2 supergravity theory and $\mathcal{N}$=4 super Yang-Mills theory are similar. In another word at mentioned limit we obtained results of ref. [17] (see table 2).

Finally by considering higher derivative terms (first order correction) in solutions (3.2) we obtained effect of such terms in drag force on quark in $\mathcal{N}$=2 supergravity thermal plasma. We found that at $\eta \to 0$ limit, the drag force in various situations is generalized by replacing $\Lambda \to \Lambda_{\rm eff}$.

For future works it is interesting to investigate $\eta \to 0$ limit in other calculations such as jet quenching parameter [38 − 45] or shear viscosity [46 − 48]. In that case one may find correspondence between $\mathcal{N}$=2 supergravity and $\mathcal{N}$=4 super Yang-Mills theories.

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Quantity	$\mathcal{N}$=2 supergravity	AdS
Minimum radius of D-brane	$\sqrt{\left(\frac{A_+}{T_0}\right)^2 + \frac{2A_+}{T_0}\sqrt{\frac{3Q}{2}} + \frac{Q}{2}}$	r_m
Radius of horizon	$\sqrt{\left(\frac{B_-}{T_0}\right)^2 + \frac{2B_-}{T_0}\sqrt{\frac{3Q}{2}} + \frac{Q}{2}}$	r_h
Lagrangian mass	m	$T_0\left(r_m\sqrt{1+\frac{Q}{r_m^2}} - \sqrt{Q}\right)$
Thermal rest mass shift	$\Delta m(T)$	$T_0\left(r_h\sqrt{1+\frac{Q}{r_h^2}} - \sqrt{Q}\right)$
Static thermal mass	$M_{\text{rest}}(T)$	$T_0\left(r_m\sqrt{1+\frac{Q}{r_m^2}} - r_h\sqrt{1+\frac{Q}{r_h^2}}\right)$

Table 1: AdS/CFT translation table. Static thermal mass of quark, $M_{\text{rest}}(T)$, is equal to free energy of rest quark in $\mathcal{N}$=2 supergravity plasma which is Lagrangian mass of quark m at the limit of zero-temperature. We see that shift of thermal rest mass $\Delta m(T)$ is depend on r_h. Here we define $\eta \sinh^2 \beta \equiv Q$, $M_{\text{rest}}(T) + \Delta m(T) \equiv A_+$ and $m - M_{\text{rest}}(T) \equiv B_-$.

Quantity ($\times L^{-2}$)	$\mathcal{N}$=4 SYM	AdS
Minimum radius of D-brane	$\frac{M_{\text{rest}}(T)+\Delta m(T)}{T_0}$	u_m
Radius of horizon	$\frac{m-M_{\text{rest}}(T)}{T_0}$	u_h
Lagrangian mass	m	$T_0 u_m$
Thermal rest mass shift	$\Delta m(T)$	$T_0 u_h$
Static thermal mass	$M_{\text{rest}}(T)$	$T_0(u_m - u_h)$

Table 2: AdS/CFT translation table for heavy quark through $\mathcal{N}$=4 super Yang-Mills thermal plasma. Table 1. At the $\eta \to 0$ limit and rescaling $r = L^2 u$ reduces to the table 2.

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